UNCERTAINTY IN ECONOMICS: AN EXTENSION OF KEYNES' ANALYSIS

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DECLARATION

I hereby declare that the material in this thesis has not been submittted to any other institution for the requirements of a degree, and is of my own composition.

John C. Peterson

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ABSTRACT

Much of the content of Keynes' General Theory of Employment, Interest and Money has not been carried through into the mainstream of Keynesian economic thought. This is particularly so with Keynes' analysis of the role of expectations and uncertainty in the determination of the 'state' of the economy.

It is argued that a major reason for the omission of this component is the necessary indeterminance of any model in which expectations are an independent, and yet deterministic, variable. By arguing that a capitalist economy operating under conditions of uncertainty must be analysed in terms of a periodic model, in which the underlying decision functions are non-monotonic, it is demonstrated that a conceptual model can be constructed which retains the essential features of Keynes' analysis even though incorporating expectations into the model as an endogenous variable.

In the process of developing this model, the characteristics of difference and differential equation systems are analysed and it is shown that the existing understanding, and therefore usage, of such systems in economics is incomplete.

Having set out a deterministic model of an economy incorporating expectations and uncertainty, the implications of the existence of uncertainty for the role and scope of economics are then considered. The conclusion reached is

that, given the conditions of the real world, the scope for accurate long-range prediction of the state of the economy is very limited.

INTRODUCTION

The General Theory of Employment, Interest and Money[1] (Keynes, 1936) has generated two waves of interpretation in the economic literature. The first wave, occurring in the years immediately following publication including works by Hicks (1937), Hansen (1953) and Dillard (1958), gave rise to the standard IS-LM interpretation which formed the basis of what subsequently became known as 'Keynesian' economics. The second wave, initiated by Leijonhufvud (1968), Davidson (1972) and Minsky (1975) subsequently sought to explain what Keynes 'actually' said and, in particular, sought to contrast this with the 'Keynesian' interpretation (Leijonhufvud, 1968, p.6).

Aim

While this thesis recognises the significance of the need to re-interpret Keynes' analysis, and is in many ways consistent with this criticism of the Keynesian interpretation, its purpose is more than mere critique. The aim of this thesis is to extend the economics of the General Theory by closing a theoretical loop-hole in Keynes' analysis of the role of expectations in an economy operating under conditions of uncertainty. This difficulty stems from Keynes' specification of expectations as both a deterministic variable and a fully independent variable (1936, p.246-7).

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1. Hereafter called the General Theory

The implication of such a specification is that the future path of the economy is indeterminant, both in the medium and long term. The response by many economists to this specification has been to ignore the deterministic features of expectations either explicitly, as in the IS-LM analysis which assumes expectations to be fixed, or implicitly, as in 'rational expectations' which assumes expectations to be 'correct'.

Thesis

This thesis seeks to show that a model can be constructed, at least on a conceptual level, which retains expectations as a deterministic variable, and therefore remains consistent with Keynes' analysis, but which incorporates expectations as an endogenous variable. In addition, it can be shown that the very long-run 'state' of the economy is not fully indeterminant though the states applying in intermediate periods are.

Form of the Model

It should be emphasised that the model constructed below is 'conceptual' and does not attempt to specify precise quantitative relationships. This method of development is followed for two reasons. Firstly, the results of the model are so critically dependent on the exact specifications of the functional relationships outlined that, for all practical purposes, the development of a quantitative version of the model is impossible. The second, and more important reason, is that advanced by Keynes (1973b, p.299), namely that any quantitative specification of a model is certain to be incorrect in the following period and serves merely to detract from the model's usefulness as an instrument of thought.

Definitions

The terms 'Keynesian economics' and 'Keynes' economics' are widely used throughout this thesis. The definition of 'Keynesian' theory used is that put forward by Leijonhufvud (1968, p.4), being that of the body of thought constituting mainstream economics. This definition therefore encompasses both the traditional Keynesians of the IS-LM analysis school, as well as the monetarist/new classical school. By Keynes' economics is meant the theory presented in the General Theory and subsequently elucidated in the articles and discussions presented in the Collected Writings of J.M. Keynes, Volumes XIV and XXIX (Keynes, 1973b and 1979).

GENERALITY IN ECONOMICS: METHODOLOGICAL FOUNDATION

A proposition which would be readily accepted by the majority of economists is that the concept of 'generality' has a significant place in economics. Some examples of the wide usage of 'generality' in economics are:

(i) In the General Theory the opening sentence reads: "I have called this book the '<u>General Theory of Employment, Interest and Money</u>', placing the emphasis on the prefix <u>general</u>." (Keynes, 1936, p.3)

(ii) For Harry Johnson Keynes' General Underemployment Equilibrium becomes a "special case of dynamic disequilibrium" (Johnson, 1961, p.13) "depend(ing) on wage rigidity or on a special empirical assumption about the monetary consequences of wage changes" (Johnson, 1961, p.3).

(iii) In present times a modern text-book writer, when comparing Keynesian and Monetarist models of exchange rate determination, concludes that "the Monetarist equation turns out to be a special case of the Keynesian equation" (Argy, 1981, p.268).

It would appear then that these economists perceive some advantage or superiority in a theory being 'general' in the sense used above. This perception stems from the tendency of economists to view more general theories as being more complete or applicable, and in some way 'better' than a more restricted or special case theory.

Certainly one of the claims to significance that Keynes made for the General Theory was that it was more 'general' than the classical theory it sought to replace. Equally clearly, one of the main forms of attack on the General Theory has been to show that it is a special case of the more general neo-classical theory.

The clearest example of the inadequacy of using the concept of generality to differentiate between theories is, of course, contained in the two examples of its use just given. How can Keynes validly claim, and demonstrate, that the theory presented in the General Theory is more general than classical theory, while the neo-classicists can validly demonstrate that Keynes' theory is a special case of classical theory? The answer, of course, is that both are correct in their arguments, while being equally incorrect in believing that the arguments have any significance as a valid basis for comparing theories. This error, made by many economists since Keynes, has been a significant element in the development of Keynesian theory. It has also contributed to the acceptance by many economists of logically valid, but inapplicable, theories for use in policy formulation.

To demonstrate the inapplicability of 'generality' as a basis for comparing theories, let us remain with the Keynes/Neo-Classical example given above and consider the use of assumptions in these arguments. Keynes argues (Keynes, 1936, pp.15-16) that the classical economics represents the restricted, full employment case of his more general theory. In other words, Keynes' argument is that the classical economists impose the assumption that full employment always holds. The neo-classical economist, for example, Johnson (1961, p.3) would argue, however, that Keynes' theory is a special case of classical theory in which an assumption of wage rigidity is made.

One point that is obvious from this example is that differences between theories perceived, as a result of the way in which assumptions are presented, are purely semantic and cannot form the basis for arguments about the generality of theories. Thus, statements such as;

> Keynes' theory is a special case of neo-classical theory in which the assumption of perfect labour market information is relaxed,

and

> Keynes' theory is a special case of neo-classical theory in which the assumption that wages are sticky is made,

while implying the relaxation of an assumption in one case and the imposition of an additional assumption in the other, in fact specify identical theoretical structures. Therefore, statements such as these, and the alternative statements describing neo-classical theory as a special case of Keynes' theory, do not, in fact, say anything about the relationship between the two theories, apart from the simple fact that they are different.

What can be concluded from this example is that both Keynes' theory and Neo-Classical theory are special cases of each other, in that by the imposition (or equivalently relaxation) of an assumption the theoretical structure of one can be changed into the theoretical structure of another. Equally, either theory can be altered into any other theoretical structure by the alteration of one or more assumptions. It can therefore be validly argued that any theory is a special or limiting case of any other theory. Equally, it can be argued that any theory is more general than any or all other theories.

The simple example given above implies that an analysis of the assumptions on which theories are based does not provide a basis for comparison. More precisely, comparison of assumptions does not allow us to determine whether one theory is more 'general' than another. Thus all arguments which attempt to establish, either explicitly or by implication, the superiority of one theory over another on the basis of 'generality' are necessarily vacuous.

Having demonstrated that one of the most common uses of the term 'general' in economics is essentially invalid, it should now be pointed out that there is a usage which is valid. This is the usage implied when we refer to 'General Equilibrium'. It also applies to Keynes' theory. These theories are 'general' in a way that fundamentally differs from the case discussed above. 'General' here refers to the aggregation of a series of specific theories (e.g. of output, prices, etc.) into a unified whole <u>within which all underlying assumptions are the same</u>.

General Equilibrium theory represents a generalisation of single market optimisation and equilibrium into the case for many goods, all within the framework of the one set of assumptions. Similarly, Keynes' theory is a 'general' theory in that it combines theories of production, investment, consumption, money and prices, all based on the one set of assumptions.

Having outlined the methodological issue being discussed by means of the Classical/Keynes debate, it will now be presented in a more general form. Most sciences, including both the Social sciences such as economics and the Physical sciences, can be considered to apply to a series of 'universes'. The characteristics of these universes are defined by their parameters. In the Physical sciences the parameters are well defined as being those of the world around us. This definition is unique because of the relative exactness with which the parameters of the real world (e.g. the speed of light, gravitational forces, etc.) can be measured. Thus the science of physics has tended to focus exclusively on a single universe defined by the parameters of the real world.

Economists are not fortunate enough to have a well defined and fixed set of parameters within which to construct their theories. As is well known, there are few measures in economics which could be called 'precise' when talking about one particular economy. In addition, there is the complication that economic agents operate in many different countries, each with their own set of laws and natural characteristics. Thus in the real world there are a range of different types of 'universes' within which the economist could decide to construct a set of theories. As a further complication in economics, we tend not to be constrained to restricting our theorising to universes that are found in the real world, and in fact one of the most influential methodological essays in economics suggests that the less realistic the universe the better will be the resultant theorie. (Friedman, 1953, p.14).

Thus economists have constructed a series of theoretical universes, based on different assumptions about the real world, within each of which there are theories of more or less generality. General Equilibrium theory and Keynes' economics are the 'general' theories in two of these universes.

A key point established previously is that if you change the assumptions of a theoretical structure which corresponds to one universe, so that its assumptions are the same as the parameters of another universe (as is the case implied by a statement such as; If you impose the assumption of a sticky labour market to neo-classical economics, you get Keynesian economics), what you are actually doing is converting the original theoretical structure into that applying to the second universe. Thus you have entirely changed the nature of the theoretical structure from one to the other and all that has been demonstrated is that the two structures are different, not that one is a special case of the other.

Having demonstrated how the concept of 'generality' has been widely misunderstood by economists, it is now possible to consider the significance of this methodological proposition. The first conclusion which can be drawn is that, where comparisons of economic theories have turned on the question of 'generality', then these comparisons, and the conclusions drawn from them, are now open to question. This then provides the justification, from a methodological viewpoint, for the re-surveying and re-analysis of the critical arguments which resulted in the transformation of Keynes' economics in the General Theory into modern 'Keynesian' economics.

The second conclusion is that the realisation that theoretical structures are uniquely identified by their assumptions, and therefore distinctly different if an assumption is altered, weakly lends support to the view that the realism of a theories assumption is significant. To this extent, therefore, this analysis serves as an alternative point from which to view the methodological proposition put forward in Friedman's "Methodology of Positive Economics" (1953) and therefore might provide the basis for a counter argument to this proposition. It should be emphasised, however, that by itself this is not a counter argument.

In general, the conclusion of this chapter is that all economic theories can be shown to be limiting, or special, cases of any other theory. Thus the argument that a theory is 'superior' to another on the grounds of 'generality' should be rejected. The point of significance of this in this thesis is that, as the assumptions which underly any theory are equally as valid as those underlying any other theoretical structure, then the use of Keynes' economics as the starting point for the remainder of this thesis is not open to methodological question. Nor can it be argued that modern, neo-classical theory provides a 'better' starting point from which to consider the activity of an economy operating under conditions of uncertainty.

THE BEHAVIOUR OF ECONOMIES UNDER CONDITIONS OF UNCERTAINTY

Keynes' economics, as presented in the General Theory, consists of two distinct components. Firstly there is the static analysis of the determination of equilibrium levels of output, income, consumption, savings, investment, prices and interest rates. This analysis is of an economy in which there is a degree of uncertainty, but constant expectations. An evolved form of this model, usually minus the uncertainty, is well known to modern economists via the Hicks-Hansen IS-LM diagram and the subsequently developed aggregate supply and demand analysis presented in texts such as Dornbush & Fisher (1978). In effect, this part of Keynes' model gave rise to the key features of modern 'Keynesian' economics. The second component is the higher level of analysis in which there is uncertainty and variable expectations. In this type of model the economy never reaches an 'equilibrium' position. This part of the analysis has been largely ignored by the Keynesians, who built their theories on what is essentially an analysis of the equilibrium positions given by the static model. The aim of this chapter then is to follow up the theoretical analysis of economies operating under uncertainty with a view to setting out their essential characteristics.

Uncertainty

The main point to emphasise is that the concept of 'uncertainty', as used by Keynes in the General Theory, differs markedly from the notion of 'risk' commonly utilised by economists when dealing with uncertain situations. Essentially, a 'risk' is a probabilistic concept in which various possible outcomes are assigned probabilities which sum to one. This requires that some form of probability distribution can be calculated for a situation, and from this an 'expected' result can be obtained.

The point that Keynes (1936), Knight (1933) and Shackle (1955) have attempted to make is that in many situations in economics the probability of a particular result occurring cannot be measured. In fact, it is usually the case in the real world that information about the future is incomplete or non-existent. Under this type of uncertainty it is not possible to assign probabilities, nor is it possible to act in the knowledge that, over time and through multiple repetition, the 'expected' result will be attained on average.

This then raises the other point which goes hand in hand with uncertainty, namely that many decisions are not reversible, or not at least without great cost (Shackle, 1956, p.6; Davidson, 1972, pp.15-16). Thus restoring an initial position, so that a particular decision (experiment) can be repeated over and over again, may be impossible. In this case, even if the expected outcomes conform to some type of probability distribution, there is no way that the 'expected' result will be attained in the long run due to repetition. Thus decisions taken in the past under conditions of uncertainty, combined with the actual results attained, determine the conditions under which today's decisions must be taken, again under conditions of uncertainty.

Of course, the effect of past decisions on today's range of options would be negated if the assets of economic agents were transmutable (putty). However, in the real world, assets, particularly capital goods, are not transmutable due to both their physical structure and the lack of secondary markets for all capital goods. This, in fact, in turn provides the basis for Keynes' speculative motive for holding financial assets. Financial assets are more transmutable than assets held in the form of capital goods and therefore widen the range of potential choices available to the economic agent.

Even the existence of secondary markets for all capital goods will not remove the uncertainty from the economy, as at all points of time someone will own each piece of capital equipment and shall face uncertainty both as to the value of the capital good in the future (i.e. the discounted expected value of all future production) and the value of the output which could be produced by the capital good in the current period. This uncertainty is twofold: firstly, as to demand (and therefore as to sale price and change in inventory) and, secondly, as to the costs of production. These costs are mainly the cost of working capital and depreciation of equipment due to usage but also includes the uncertain effects of today's production on future demand. For example, a failure to produce in the current period may depress demand in future periods due to a decline in consumers' product awareness.

Removal of the Effects of Uncertainty

There are many methods used by economists to reduce or exclude the effects of undertainty in their theories. The easiest method, and one frequently used, is simply to assume that uncertainty does not exist. Equivalently, it may be assumed that the likelihood of a particular outcome occurring conforms to some frequency distribution, and that the event occurs a very large number of times from identical starting conditions. This requires that the frequency distributions of all possible events are somehow definable and known by all participants in the economy. The effect in this second case is that all economic agents will act according to the mathematically 'expected' outcome and will, over an infinite number of repetitions of an identical experiment, attain that mathematically expected result. The effect, therefore, is that the expected outcome is held with absolute certainty by all economic agents, and further that that expected outcome is in fact the correct outcome to expect. These types of assumptions underly many General Equilibrium type models. In these models the economy attains a unique equilibrium position which is maintained indefinitely through an infinite number of identical future periods.

Another method of eradicating uncertainty from the analysis is to assume the existence of a tatonnement process by which a 'Walrasian Auctioneer', or the process of re-contracting, ensures that all prices are determined prior to the commencment of production. The usual assumption is that this process will establish a price vector which is a market clearing set of prices. The tatonnement process, of course, only alleviates the uncertainty connected with 'what to produce', given the existing capital equipment, by effectively removing all uncertainty about the results of present production decisions. It does not, however, resolve the question of uncertainty in the value of capital goods carried through to the following period. This difficulty is usually resolved by assuming either that all capital is created and then consumed within the production period, or that 'capital' is malleable. Under the assumption of malleability, the given 'quantity of capital' (however measured) is formed into the desired, return maximising, forms of capital required on the production day, and then converted back into a formless and malleable globule for use in subsequent periods.

Conceptually this is the same as assuming that initially all 'capital' is held as financial capital or financial assets in Economy A, and that there exists a market for capital goods <u>external</u> to Economy A which stands ready to supply and repurchase infinite quantities of all particular capital goods at <u>fixed</u> prices in any production period. Thus agents in A will purchase the capital good required in the current period at known prices and will then resell, the now used, capital equipment to someone <u>outside</u>

the economy at the end of the period, again at known prices. The effect of this is to reduce uncertainty as to changes in the value of capital between the beginning and end of one production period to zero within Economy A.

Even this device of putty capital, which ensures that no agent in A is disadvantaged by changes in the relative values of capital between periods, will not fully alleviate uncertainty in a multi period economy. This is because the owners of capital will not know what return will be derived from their liquid capital in the following and subsequent periods. They will therefore be unsure as to how much, if any, of their capital should be consumed in the current period and how much carried through into the future for use in subsequent production. There are several ways to remove this uncertainty, or of ensuring that it has no effect. The trivial case is to assume perfect information about the future. This is trivial as the net effect of tatonnement in the current period, and perfect future information is to return us to the 'complete perfect information' case which simply assumes away uncertainty.

The second alternative, which is utilised by the 'Steady State' Walrasian General Equilibrium model, is to assume that the value of capital is maintained between periods. That is, the producer has no choice as to the quantity of capital which he holds and must always assign to his liquid assets the monetary value of the difference between the purchase price of capital assets at the beginning of the period and their resale value at the end of the period, i.e. the amount of depreciation. For this to fully alleviate uncertainty, it is also necessary to assume that the prices at which capital is purchased and resold does not alter from one period to the next, thus maintaining the value of capital, relative to other assets across periods. The effect of these assumptions would be to ensure that, as factors are unchanged, the activity of the economy will be identical in all periods. This assumption is, in effect, identical to the assumption that all capital is produced and consumed during the one production period.

The condition that each owner of capital maintain his level of liquid capital from one period to the next can be relaxed if we allow that the liquid capital of the economy is identical to the quantity of savings of the economy, and that the rate of interest will act to ensure that the required quantity of savings if forthcoming. Therefore, if one producer wishes to decrease the quantity of capital held during the period, others will be tempted, by an infinitely small increase in the rate of interest, to increase savings so as to acquire the necessary quantity of liquid capital. For this condition to apply, however, it is necesary that all future periods be identical to the present so that the present yield on capital is equal to the expected yield. If this were not the case, then uncertainty about future yields could give rise to the possibility of the supply of savings differing from the desired amount of investment.

The third alternative is where the rate of change of population or technology is not assumed to be zero and is such that constant increases in the quantity of capital will maximise individuals welfare and therefore producers returns. Providing that either all information about the future is known, or that all future periods are the same as the previous period inflated by a known growth rate, then all uncertainty will again have been removed.

These three cases are essentially the same as assuming perfect knowledge for the current period and an infinite repetition of that period into the future, possibly with known or predictable changes of scale. Thus again the future is fully resolved and no uncertainty enters into the model.

A subtle variation on this theme is the relatively recently developed literature on 'Rational Expectations' (for example, Muth (1961), Sargent and Wallace (1975)). This assumption drops the need for all future periods to be identical to the present for the 'expected' results to be attained in the longrun. Instead, rational expectations requires that all economic agents assign a probability of one to the occurrence of one particular outcome. In addition, this outcome must happen to coincide with the actual outcome so that transactions entered into on the basis of an expectation held with certainty do not lead to unexpected results.

This was essentially the view, of agents acting in markets, held by Keynes when discussing the concept of a 'conventional' valuation of an asset (Keynes, 1936, pp.152-154), with the exception that he admits the possibility of the conventional valuation at any point in time being incorrect, and therefore subject to significant fluctuations, given the receipt of additional information. In the rational expectations world the conventional valuation must be correct and the possibility of new information altering the expected result is not allowed. Again then, we have reverted to the assumption that all information is known by all agents.

It is interesting to note that an assumption essentially the same as rational expectations was made by Keynes to make exposition of the model presented in the General Theory easier. Under rational expectations there is no change in information which would cause agents to alter expectations. In setting out his static model, Keynes assumes that the marginal efficiency of capital schedule is fixed, and therefore that the receipt of new information does not cause producers to revise their expectations of return on investment. This assumption was subsequently dropped in Keynes' description of his dynamic model in Chapter 22 of the General Theory where he discusses the trade cycle.

An alternative method of removing uncertainty from the analysis is to allow for the existence of contracting through time. This potential method of decreasing uncertainty allows for two possibilities, only one of which completely removes uncertainty.

The first possibility, which is that utilised in the Arrow-Debreu model, is to allow for the recontracting of all futures contracts prior to the commencement of production in the first period. The result then is essentially the same as for the single day operation of an economy with a Walrasian Auctioneer, in that all future values of all assets are determined today and contracted for. This method allows future periods to differ from the present, however it does require knowledge of all future asset supplies, production technologies and individual preferences. (This does have the interesting requirement that, not only do economic agents need to know the preferences of all unborn people, they also need to be able to enter into contracts on their behalf.) The other requirement here is that futures markets must exist for all assets (noting that there is no place in this model for a numeraire). Should even one futures market fail to exist then the net effect will be to retain all of the uncertainty facing the economy and concentrate it onto that particular asset and time period for which there is no futures market. Clearly again, the result of assuming that Arrow-Debreu type futures contracting is feasible is equivalent to assuming away all uncertainty.

It could be argued that contracts are methods of significantly reducing uncertainty. Contracts do not reduce uncertainty, unless they pass it outside the economy as was the case of the external capital purchaser above, they merely transfer the uncertainty to those most willing to bear it. In this way contracts can reduce the significance of uncertainty, but can never eliminate it, except in the Arrow-Debreu case.

On the face of it, the activity of insurance may appear to lead to a reduction in uncertainty via a spreading of risks. Insurance, however, is limited to those situations in which the likelihood of an event occurring can be assigned actuarial probabilities summing to one. Thus, while insurance is applicable to instances such as acts of nature which are random with some identifiable distribution about a mean, or Arrow-Debreu states of nature which conform to some known probability distribution, this idea cannot be used to remove uncertainty from business decisions in which there is a fundamental lack of knowledge on which to base expectations about the future. Another significant difference between insurable situations and business decisions is that for insurance to work, the ability of the insured to influence the outcome must be essentially zero. This condition is not satisfied in business decisions where the actions of the entrepreneur will influence the outcome of any situation. Crucial decisions, in Shackle's sense (1955, p.6), are a fundamental feature of the operation of an economy and are diametrically opposed to the 'certainty' or 'insurable risk' assumptions made in economics.

Having identified the various conditions necessary for the effects of uncertainty to be safely excluded from the analysis of economic activity, the assumption is now made that these conditions do not apply. The remainder of this chapter will therefore proceed to consider, and set out, the characteristics to be expected of an economy in which uncertainty and the state of expectations are significant deterministic variables.

Necessary Conditions

The first requirement is that there not be perfect information, and that uncertainty as to the 'state' of the economy, in both the present and the future, should exist in the minds of economic agents. Under true uncertainty, as opposed to risk, different economic agents will hold different expectations as to the future state of the economy. This will occur because agents possess different information and utilise different methods of interpreting that information based on different prior experiences. Furthermore, these expectations will be held with varying degrees of certainty, or confidence (Keynes, 1936, p.148). Generally, therefore, an economic agent will form a particular expectation about the future but will not hold that expectation with certainty. In fact, an agent is likely to believe that the expectation held at any one point in time about any future time will prove to be incorrect. Thus expectations are likely to be variable, depending on the receipt of new information by the agent (Keynes, 1936, p.315).

The second requirement is that expectations must be significant, i.e. they must be able to affect the state of the economy in the futue. For this requirement to hold, two conditions must be satisfied:

(i) Expectations about the future must be able to influence actions taken in the present.

(ii) Actions taken in the present must be able to affect the state of the economy in the future.

To satisfy the first of these conditions expectations must be able to influence the decisions of economic agents, and therefore affect the state of the economy in the present period. Thus there can be no independent feature of the economy (such as labour market clearing via fluctuations in real wages) which will cause the economy to adopt a particular state irrespective of agents' decisions. If this were the case then the possible 'states' of the economy would be fully determined into the future and there would be no uncertainty. A corollary of this first condition is the requirement that the economic system have extended existence, i.e. it must be expected to exist for an extended (infinite) period of time. This requirement is related to the existence of money. As is obvious from consideration of the Arrow-Debreu model, or of any economy in which there is no uncertainty, there is no room for money in an economy without uncertainty (for example, Knight, 1935, xxii; Samuelson, 1963, pp.122-4; Keynes, 1937, pp.115-6; Davidson, 1972, pp.140-147). Equally, there is no room for uncertainty in an economy without money. This can be seen by appreciating that an economy without money will necessarily be a barter economy, and therefore constantly at 'full' employment. If an economy will continuously be in full employment through time, the ability of expectations about the future to affect the state of the economy, either in the present or in the future, will be zero.

Thus we have the situation of no money without uncertainty and no uncertainty without money. The existence of money, however, is dependent on the expectation that it will retain its value into the future. As noted by Hahn (1982, pp.2-6), it is necessary for economic agents to expect that the economy will exist for an extended period of time for money to have value, and therefore exist, today. If the economy were not expected to exist for an extended period of time, then the terminal state of the economy would be known. The exchange value of money in that last period will be zero as no-one would be prepared to exchange anything with intrinsic value, such as goods for money which has no intrinsic value. In a rational economic world this will cause the value of money in the second last period also to be zero, as no-one would be prepared to accept money in this period and hold it through into the final period. A simple application of induction will show that the value of money in <u>all</u> periods must be zero if a 'final' period for the economy is envisaged (Hahn, 1982, p.5). Thus the three conditions of existence of uncertainty, existence of money and expectation of the existence of the economy over an infinite number of periods are mutually dependent. The significance of this then is that it is necessary to consider the activity of an economy operating under conditions of uncertainty in terms of a multi-period, or sequence, model, and it is the expectation of the existence of those future periods which allows money to exist and therefore allows expectations about the state of the economy in those future periods to influence decisions made in the current period.

The second condition necessary for expectations to be significant is that decisions made in the present period should affect the state of the economy in future periods. This condition is fulfilled by the two links between the present and the future, namely accumulated production capacity (capital) and accumulated consumption capacity (savings). As has been seen above, if capital is malleable or is maintained at a certain quantity across periods, the effect is to remove uncertainty (or at least its effect) from the model, thus a necessary condition for expectations about the future to affect the future is that the decision to acquire capital equipment should be made independently by agents, and that capital, as both equipment and stocks, should be carried from one period to the next in immutable form. The effect of this then will be for the investment decisions of agents today to restrict the set of possible production decisions available to the owners of capital tomorrow. In this way then physical assets, through their expected values, become a significant link between decisions made in the present and the uncertain future.

The quantity of saving will cause decisions made in the present to affect the future to the extent that future decisions to decrease savings will affect the future state of the economy. For this effect not to be zero, the real value of savings, or at least some part of it, must be maintained across periods, and not destroyed by completely offsetting changes in price levels in future periods. Thus a further requirement for expectations to be significant is that inflation should not completely make the level of demand in money terms irrelevant in all periods. The corollary of this is that saving decisions in the present period should affect the level of demand in the present period and should not be fully offset by changes in the price level. The impliction of this is that a multiplier, even if of less than unity, must exist in an economy operating under uncertainty. If the multiplier were zero this would imply that the real value of savings, and therefore of money, was zero, and we would quickly revert to a barter type economy with no uncertainty.

Characteristics

From the above analysis of the conditions necessary for the two essential requirements of an economy operating under uncertainty to hold, we can derive the following expected characteristics of a non-static economy:

(i) It must necessarily be viewed as periodic or sequential.

(ii) The 'state' of the economy in any period is related, although not in a fully predictable way, to the decisions made by agents in the previous period. The influence of these decisions are transmitted from one period to the next by:

(a) the types and quantities of capital equipment;

(b) the types and quantities of stocks;

(c) the quantity of financial assets.

(iii) The movement of the economy through time is essentially unknowable, being dependent on the unknown outcomes of decisions made under conditions of uncertainty.

(iv) While movements between any two periods (whether coincident or not) are unpredictable, the 'states' that the economy does attain must be constrained within certain limits required for the maintenance of the institutions of the economy and of the political parameters within which it operates.

At this point we can outline, essentially as derived from Chapters 5, 18 and 22 of the General Theory, the major characteristics of a capitalist economy operating under conditions of uncertainty and without active government involvement in the economy's operations.

In a capitalist economy the economic agents are households and firms. The main activity carried out by these agents is decision making. Households make decisions about the proportions of their incomes which they save or consume. Firms make decisions about how much to produce in the current period, given the existing quantity and type of capital equipment, (i.e. decisions as to how much working capital to invest) and about how much new capital equipment to acquire (investment in physical capital). At the most fundamental level then, the quantity of output will depend on the decisions of firms and households. These decisions will be based on uncertain expectations about an uncertain future. Thus the level of output at any time in the future will be indeterminate. We could expect, however, that as expectations are likely to undergo waves of varying strength and duration, of optimism and pessimism, then the level of output would be expected to vary in a wave-like or cyclical manner. The nature of this cyclical path, however, being based on expectations formed from information that is not known in the present period, will, for all intents and purposes, be essentially random in that the probability of the economy conforming to any particular state at some point of time in the future is indeterminate.

While the above description provides a general outline of the expected behaviour of capitalist economies operating under conditions of uncertainty, it does not provide, even at a conceptual level, an analytical tool for such an economy. Thus the tendency in economic thinking has been to fall back on deterministic models of the economy in which the economy does not wander off into an indeterminate and unanalysable future. Such models clearly exclude some essential characteristics of the short-run behaviour of economies operating under uncertainty, and would not, therefore, necessarily model the long-run characteristics accurately either. In fact, the available data would seem to suggest that the real world economy does not approach, and remain at, a steady state equilibrium condition. Instead, it appears that short-run trade cycle type behaviour, around some long-run level, might be a more realistic description.

Variables

In describing the operation of an economy operating under uncertainty, it will be necessary to identify those variables which best characterise the 'state' of the economy, and which also are significant determinants of the state of the economy in the following period. In Keynes' model the key determinants of the decisions made by agents in one period, which determine the state of the economy in the following period are: the 'state' of expectations, the level of output and the distribution of income between workers and entrepreneurs.

Expectations operate directly on the investment, production and consumption decisions of agents by being major determinants of the schedule of marginal efficiency of capital, the liquidity preference schedule and the propensity to consume. The existing level of output and the related level of employment serve to determine the levels of stock and excess capital equipment held, as well as influencing the level of real wages. The level of output therefore, would influence all agents in their decision making. The distribution of income is not usually identified as a significant determining variable, however it is likely to be a significant determinant of the levels of production and investment in any period. This can be seen from considering the extreme situations in which the income distribution strongly favours firstly workers and then capitalists. Where workers' share of output is large, a decline in this share may be expected, all other things being equal, to increase the expected return on investments and therefore lead to higher levels of investment and output. Alternatively, where labour's share of incomes is low, an increase in this share may be expected to decrease the savings ratio significantly and thereby lead to an expansion in the level of output.

Functions

Having identified the key explanatory variables as being expectations, the level of output and income distribution, and recalling that the essential characteristic of economic agents is decision making, it is clear that the fundamental functions determining the future 'state' of the economy, in terms of expectations, level of output and income distribution, will be the response functions of agents to the existing 'state'. Mathematically, this would be represented by the function:

$$(E(t), U(t), D(t)) = f(E(t-1), U(t-1), D(t-1)) \quad (2.1)$$

where E = level of expectations

U = level of unemployment (%)

D = income distribution (%)

The level of unemployment is utilised here as a proxy for the proportion of potential output achieved.

Leaving aside for a moment the question of the incorporation of expectations into the model and assuming that the remaining model is continuous in time (a differential model), it is possible to specify the general shapes of the two functional relationships between employment and income distribution.

For any given income distribution there will exist an objective optimising function in output on the part of producers. Normally it is assumed that the objective to be maximised is profit. On a priori grounds, it can be argued that this function will call for a decrease in the level of output from very high output levels, and for an increase in the level of output from very low output levels. The general shape of the graph of this function will, therefore, be a concave curve with, by assumption, a single maximum in the range $0<U<1$ and values of approximately zero profit in the extremes. Focussing only on the output axis, this specifies a reaction function in employment with movements towards a particular level of output for each level of income distribution.

The income distribution function can also be expected to be concave, with economic forces likely to create a movement towards a particular distribution, for each given level of output. Again, as argued above, these distributions are unlikely to include the extreme cases of either workers or capitalists having a zero share of national income.

The general specifications of the model just outlined correspond to the model set out by Goodwin in his 1965 article (1965, pp.54-58) with the exception that Goodwin's functions are determined by structural characteristics and are not response functions to expectations. The essential feature of these models are that the response functions are non-monatonic, or folded, functions. It has already been established that the analysis of an economy operating under conditions of uncertainty must be carried out in terms of periodic, or iterative analysis, in which the results of one period are fed back into the reaction functions to determine the results of the following period. Investigation of the nature of the fundamental relations suggests that the response functions under consideration should be non-linear and, in particular, non-monatonic. Thus a model of an economy operating under conditions of uncertainty must be based on non-linear differential or difference equations.

At the present stage it has not been shown how expectations can be introduced into the model. Their formal introduction will be delayed until Chapter 4, however the method of introduction will be outlined in Chapter 3 which outlines the key features of non-linear differential equation and difference equation systems.

DIFFERENTIAL AND DIFFERENCE EQUATIONS

While the outlines of the model being developed are, at this stage, most easily thought of in differential equation form, it shall be seen that the differences between differential and difference systems are of no significance to this thesis. As the final version of the model will be presented in difference equation form, this chapter surveys the current usage in economics of difference equations, particularly non-linear difference equations, and shows how the behaviour of functions as described in Chapter 4 fits into this analysis. The relationship to differential equation systems will subsequently be made clear.

Linear Difference Equations

Difference equations, especially linear difference equations, are widely used in econometrics. The analysis of the properties of linear difference equations is quite complete (for example, Samuelson, 1963, pp.391-439 and Baumol, 1970, pp.151-253). As Baumol demonstrates, the solutions of simple first order linear difference equations of the form:

$$y(t) = f(y(t-1))$$

can be generalised into the form:

$$y(t) = AM^{t}$$

where A and M are numbers (Baumol, 1970, pp.161-162). A represents the 'initial' value of y at time t=1, while M represents the function of transformation of y(t-1) into y(t). There are then three classes of results depending on the values of A and M:

(i) A positive and M positive

a) $M=1$: value of y is stationary

b) $M>1$: value of y will increase explosively

c) $M<1$: value of y will decrease by ever deceasing amounts

(ii) A positive and M negative

a) $M=-1$: y oscillates from +A to -A indefinitely

b) $0>M>-1$: Damped oscillations

c) $M<-1$: Explosive oscillations

(iii) A negative and M positive

a) $M=1$: value of y is stationary

b) $M>1$: value of y will decrease explosively

c) $M<1$: value of y will increase by decreasing amounts

These results can be generalised into three statements (Baumol, 1970, p.164):

(a) y(t) will explode, decrease towards nothing or stay constant as M is greater than, less than or equal to 1 in absolute value.

(b) y(t) will move with or without oscillations as M is positive or negative.

(c) A change in the sign of A will turn the results upside down.

For higher order linear difference equations such as:

$$y(t) = ay(t-1) + by(t-2)$$

which is a second order linear difference equation, the results are more complicated. For this simple second order case, Baumol (1970, p.171-174) demonstrates that the solution of all second order difference equation systems will take the form:

$$y(t) = a(x_1)^t + b(x_2)^t + Z$$

where a, b and z are constants and x_1 and x_2 are the roots of an equation of the form:

$$x^2 + bx + c = 0$$

(roots assumed not to be multiple).

The various possible solutions of this type of system are:

(i) Both roots real and non-negative

(a) If both x_1 and x_2 are less than unity : y(t) asymptotically approaches z

(b) One root equal to 1 and the other less than 1 : y(t) will approach a+z or b+z dependent on whether $x_1 = 1$ or $x_2 = 1$ respectively

(c) If either or both roots exceed unity, y(t) will explode positively or negatively, depending on the sign of the co-effect on the largest root.

(ii) Both roots real, neither positive

(a) Both roots less than unity in absolute value : damped oscillations about z

(b) One root equal -1 while the other is less than 1 in absolute value : Damped oscillations down to the values z+a, z-a respectively in the case where $x_1 = -1$, or to z+b, z-b where $x_2 = -1$.

(c) Either root greater than unity : Explosive oscillations

(iii) Both roots real with one root positive, the other negative.

Here again, the general properties hold, namely that y(t) will tend to explode, remain steady or approach some particular value or pair of values, as the roots are greater than, equal to or less than unity in absolute value.

(iv) Complex or imaginery roots.

Where roots are complex, it can be shown that the values of y(t) will vary cyclically, with the lengths of the cycle varying from case to case. These cycles need not be symmetrical. Again, however, the three cases of explosive cycles, steady cycles and damped cycles are possible (Baumol, 1970, pp.206-212).

These cover the possibilities for the second order linear difference equations. Baumol (1970, p.212n) suggests that such cases will be characteristic of the higher order cases also.

Non-Linear Difference Equations

Non-linear difference equations are equations of the form:

$$y(t) = f(y(t-1))$$

where the function (f) is non-linear. Examples of such equations are:

$$y(t) = a(y(t-1))^2$$

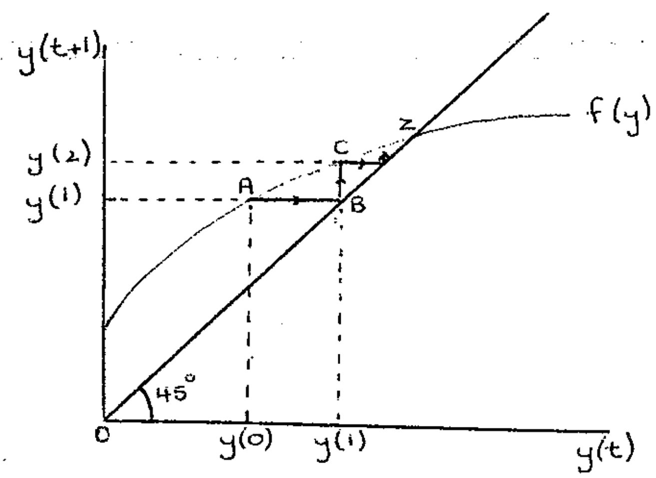
$$y(t) = \sin(y(t-1))$$

$$y(t) = 4Ay(t-1)(1-y(t-1)).$$

These example equations are all examples of non-monotonic, non-linear difference equations. That is, the values of the function increase (decrease) to a maximum (minimum) and then decrease (increase).

First order non-linear difference equation models are considered by Baumol in Chapter 13 (1970, pp.254-278). In order to aid this analysis, and so as to represent the time path of the variable (y(t)), Baumol makes use of a 'phase' diagram (1970, p.256). In the figure below, a straight line is drawn forming a 45 degree angle with the two axis labelled y(t) and y(t+1). The graph of the non-linear function f(y(t)) is also presented in this space.

Figure 3.1



Suppose that the initial value of y (the 'seed' value) is y(0) on the y(t) axis. Since y(1) = f(y(0)), the value of y(1) can be obtained from the point A, above Y(0) on the curve f(y). The value of y(1) can be transferred to the y(t) axis by moving horizontally from A to the point of intersection with the 45 degree line (B) (because along the 45 degree line both co-ordinates of a point are equal). This process can then be repeated to find all subsequent values of y(t). In this figure the time path of y(t) is moving in a stable (although asymptotic) manner towards the point Z. This time path will henceforth be referred to as the 'orbit' of y(t) and is clearly dependent on both the function f(y(t)) and the seed value y(0). It should also be noted that for this, or any other, single first-order, non-linear difference equation, the significant values (y(0), y(1), ... y(n)) can be represented as points along a line (in this cases either the y(t) or y(t+1) axis would do). Thus a single first-order difference equation will give rise to an orbit which can be represented by a series of points in a single dimension.

The possible orbits of a first-order non-linear difference equation which maintains the same general slope within the domain under consideration, are analysed by Baumol on pages 259-262. He observes that these equations can generate four basic types of orbit which are analogous to the four types of time path generated by the first order linear difference equations. These types are:

(i) Damped without oscillation

This will occur whenever the slope of the graph of f(y(t)) is positive and less than that of the 45 degree line. In this case the graph of f(y(t)) cuts the 45 degree line from above and the orbit of y(t) will converge towards the value z in ever-diminishing steps. It can be said that the successive values of y(t) are 'attracted' to a particular value z however the rate of attraction diminishes or is damped. In this case the orbit of y(t) will be attracted to z both from below (as in Figure 3.1) and from above and will not overshoot. Because the orbit will remain at z in each period after z has been reached, z is called an attractor of period one.

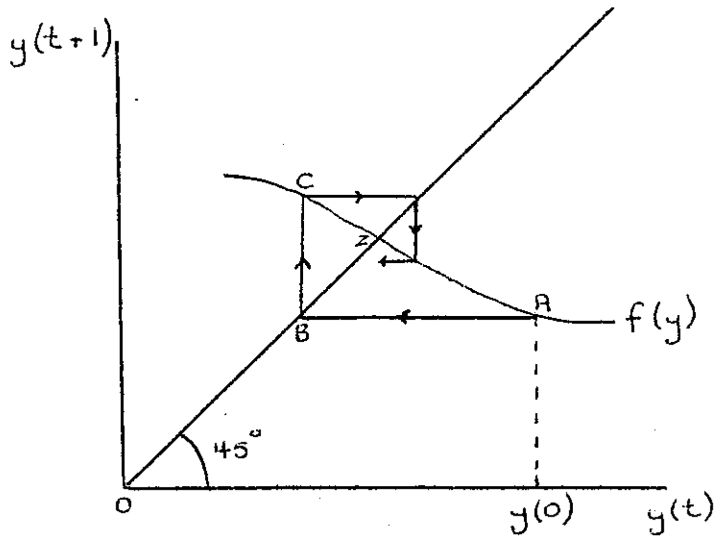
(ii) Explosive without oscillation

This path will occur where the graph of y(t) cuts the 45 degree line from below. The orbit of y(t) will be unstable and increase or decrease indefinitely as the seed value is greater than or less than the point of intersection of the graph of y(t) and the 45 degree line (point z). Note that the point of intersection is itself stable such that if y(0)=z then the orbit will remain at z.

(iii) Damped Oscillations

Here the slope of the function f(y(t)) is negative but greater than -1. The graph of f(y(t)) therefore cuts the 45 degree line from above but with an angle of between 45 and 90 degrees. Such a case is illustrated in Figure 3.2.

Figure 3.2

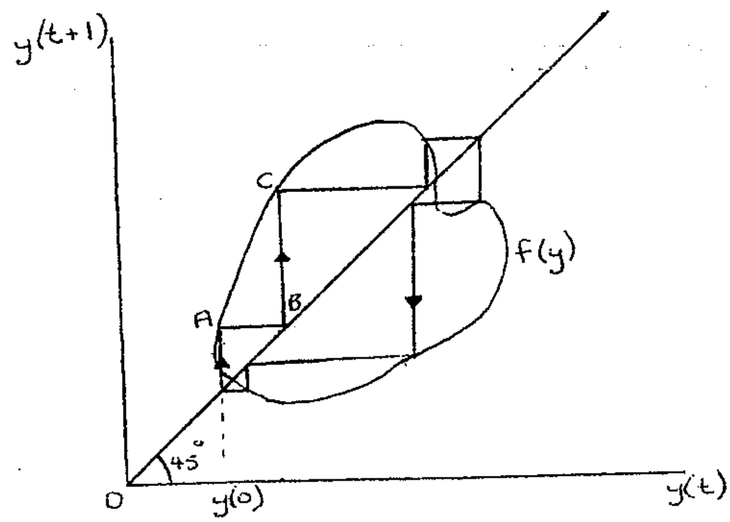


In this case each successive value of y(t) will lie alternatively above and then below the attractor (z) and the orbit will converge to this point by means of increasingly small (damped) oscillations. The point z is again an attractor of period one.

(iv) The remaining class of cases is where the function f(y(t)) cuts the 45 degree line steeply from above, i.e. the angle of intersection is greater than 90 degrees. In this case the successive values of y(t) will again oscillate around z but will continuously move further away from z. This is a case of explosive oscillation.

It should be noted that these four possible cases were derived from consideration of functions which were essentially linear in that the slopes of the functions were similar to the slope at the point of intersection with the 45 degree line at all points under consideration. Thus one form of behaviour was exhibited by the orbits considered. Baumol then goes on to illustrate how a function which varies significantly in slope along its length can give rise to an orbit which will oscillate between two or more points. In other words, the values of y(t) will be attracted towards two or more different values in succession and will produce an orbit that repeats itself perfectly after a certain number of periods. Such an orbit is termed a 'limit cycle'. A graphical example of such a limit cycle is given by Baumol (1970, p.264) and reproduced here as Figure 3.3.

Figure 3.3



While no definitive conclusion is reached by Baumol in his discussion of limit cycles, the impression is given that the oscillatory orbits of difference equations, which are neither damped nor explosive, will eventually fall into a limit cycle. The discussion is mainly of limit cycles which repeat after two periods (i.e. which are attracted to two points and which can be called an attractor of period two). An example of a higher period limit cycle is given when outlining Hicks' Trade Cycle Model (1970, pp.268-273), however this is achieved by means of a switching from one function to another.

What is argued in the following section is that this analysis is incomplete and that a general case exists in which the orbit of a difference equation neither explodes nor converges to a single value (an attractor of period one), nor does it ever form a limit cycle by repeating itself (an attractor of some period greater than one). This type of orbit is non-periodic and the series of points it generates in phase space is called a 'strange attractor'. The possible orbit paths generated by the iteration of first order non-linear difference equations can be categorised in the following table.

<u>Table 3.1</u>

- unstable (no attractor)
  - non-oscillating
  - oscillating
- stable
  - oscillating
    - Single point attractor - attractor of period 1
    - Multiple point attractor - limit cycle
    - Infinite point attractor - strange attractor
  - non-oscillating — Attractor of period 1

One of the simplest, non-linear functions is the non-monotonic or folded function. Examples of such functions include parabolas, sine waves and polynomials. For these folded functions which rise to a maximum and then decline, it has been shown (see for example May, 1976 and Hofstadter, 1981) that the orbit generated by iterating these functions as difference equations can be extremely complicated. In the following examples, in order to ensure that the value of the result will always lie within the defined input range, and can thus be re-entered into the function as an iterative input in the following period, the function is specified so that the maximum value of the function corresponds to an input value in the domain [0,1] and the value of the function within that domain lies in the range [0,1].

An example of such a folded function is:[1]

y(t) = 4Ay(t-1)(1-y(t-1)) $0<A<1$

The highest value of this function is A and occurs at the point where y(t-1)=0.5. As has been shown by Metropolis (1973, pp.39-40), choosing the maximum value as occurring at the mid-point of the input domain and restricting the domain to [0,1] does not restrict the generality of the function's properties. The pattern of values traced out by the series of iterations of this function (the orbit) has been found to depend upon the value assigned to the scaling variable A.

For values of A above 0.25 and below 0.75 the set of sequential values generated from any positive seed (as a seed value of zero will remain at zero) will converge, or be attracted to, a unique point. This unique point is called an attractor of period 1. In Tables A.1 and A.2[2] the first fifty results in the orbit obtained by iterating the function y(t) = 4Ay(t-1)(1-y(t-1)) with values of A of 0.3 and 0.7 respectively are shown. In each case, the seed values of 0.01, 0.999 and 0.5 are selected. With A=0.3 (Table A.1) the function converges to the unique value of $0.1\dot{6}$ (accurate to 16 decimal places after 500 iterations), while with A=0.7 the function converges to the attractor 0.64285 ... These are examples of simple attractors of period 1. Such attractors will occur for all values of A

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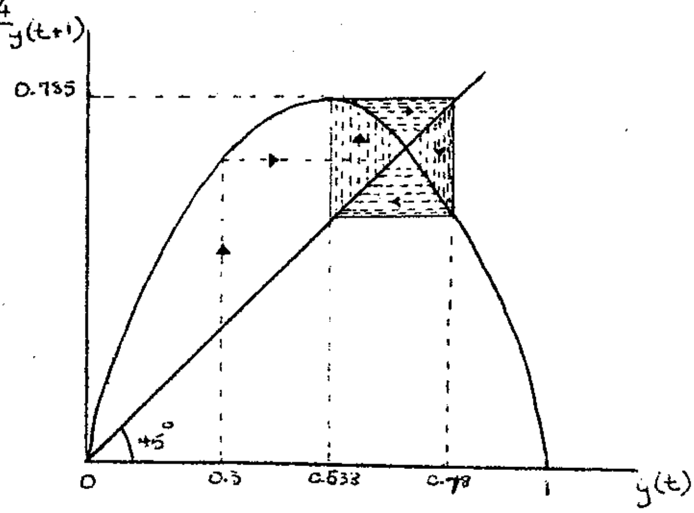
1. This example is based on that of Hofstadter, 1981, p.17.
2. Tables included in Appendix. All calculations made to 16 significant decimal places.

between 0.25 and 0.75.[3]

For values of A greater than 0.75 and less than 0.86237 ..., the iterative values of the function will be attracted, not to one point, but to two values, and will cycle endlessly between these two values. An example of this is given in Table A.3 with A=0.785 and the three seed values 0.01, 0.999 and 0.5. As can be seen, the function is drawn towards (attracted to) two distinct values 0.780464 and 0.538007. Mathematically, what has occurred is that the initial single attractor has split (as A exceeded the critical value of 0.75) into two points of attraction which move further apart as A increases. These two points of attraction are said to form an attractor of period 2 and the orbit is executing a limit cycle of period 2. An attractor of period 2 is illustrated in the phase diagram in Figure 3.4, with A=0.785 and a seed value of 0.3.

Figure 3.4



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3 For values of A below 0.25 the orbit is unstable and explodes to negative infinity.

At the next highest critical value of A (0.86237...) the two points of attraction in the two period limit cycle will split to produce an attractor of period 4. This splitting, which is called bifurcation, will occur at the two points simultaneously due to the nature of the processes by which period-doubling occurs (for example, see Hofstadter, 1981, p.19). The iteration presented in Table A.4, with A set at 0.87, shows an example of an attractor of period 4. (Convergence to the attractors was not complete after 50 iterations. After 500 iterations the attractors approached were: 0.8316801, 0.4871592, 0.8694262 and 0.3950630.)

For each higher critical value of A reached, the number of points of attraction will double, with the distance between the critical values of A becoming smaller. In fact, for this particular equation the critical values of A are asymptotically approaching 0.892486 ... More precisely, and more generally as this applies to all equations exhibiting the periodic doubling characteristic, the ratio:

$$(A_n - A_{n-1})/(A_{n-1} - A_n)$$

where A_n is the critical value at which bifurcation occurs, approaches a constant value identified by Feigenbaum (1978, p.30) of approximately 4.6692016609 ...

What then happens to the solutions of the equation for values of A greater than 0.8924864 ... and less than 1? In this region the series obtained by iteration of any particular seed value will embody an infinite number of period doublings. In this region, which is called the chaotic region, there are two possible outcomes. In the first instance the path of the series will remain periodic, however the period will be very high and the path extremely complex, making detection of the periodicy very difficult. The alternative is that the orbit path does not converge to any finite attractor. In this case the values of the path will remain bounded within some region (as the orbit is not explosive), within which the orbit will be non-periodic and, to all intents and purposes, chaotic. This type of constrained orbit is called a 'strange attractor'.

Metric Universality

Having now outlined the concepts under discussion in terms of the single equation y(t) = 4Ay(t-1)(1-y)(t-1) the more general characteristics can be set out. Firstly, the characteristics of period-doubling and decay into chaos, under the influence of a single scaling variable (A), are quite general to any smooth equation which is strictly concave downward and which possesses a single central maximum (see Feigenbaum, 1978, pp.48-49 and Metropolis, 1973, p.26). As has been observed above, all such equations converge on a final critical value of A in accordance with the equation:

$$(A_n - A_{n-1})/(A_{n+1} - A_n) = \text{Feigenbaum's Number.}$$

In addition, the pattern about the maximum value of the function generated by the attractors at the points of bifurcation (e.g. between the 4 and 8 cycles) has been found to depend on another universal constant a=2.5029078750957 ... (Feigenbaum, 1978, p.29). These universal constants, and the general nature of period-doubling and chaotic behaviour, because they depend only on the fact that a function is being iterated, and not on the particular form of the function, have been termed 'metrical universality' (Feigenbaum, 1979 and Hofstadter, 1981, p.27).

The example so far considered has been a single first-order difference equation and thus the attractors which have been produced have been able to be represented as a set of points on a single line as a single equation only involves a single dimension. It is quite possible, however, to couple the difference equations to analyse higher dimension cases (May, 1976, p.466 and Hofstadter, 1981, p.22).

For example, two coupled difference equations of the form:

$$x(t+1) = f_1(x(t), y(t))$$
$$y(t+1) = f_2(x(t), y(t))$$

will produce an orbit which would be plotted as a series of discrete points in two dimensional phase space. For systems containing n coupled difference equations, the orbit is a series of points in a phase space of n dimensions.

Differential Equations

The orbits of difference equations are discrete in that the point in phase space which represents the values of all the variables in the system jumps discontinuously from one time period to the next. In a differential system, on the other hand, the time variable is continuous and not discrete. Therefore, the path of the point in phase space which represents the value of the systems variables would form an orbit consisting of a continuous curve rather than a series of discrete points. So as to differentiate the continuous path traced out by the differential equation from the discrete path of the difference equation, the differential equations path shall be called a 'trajectory' and the term 'orbit' used to describe the path of the difference equation.

If the trajectory in phase space of the differential equation is sampled periodically, and the value of the point transcribing the trajectory recorded, then the set of sample points collected after a number of periods will form a discrete set of points exactly analogous to the discrete orbit generated by a difference equation. In fact, the orbit of an iterative difference equation is that of an iterative differential equation sampled at some 'natural' period (Hofstadter, 1981, p.22).

Natural Period

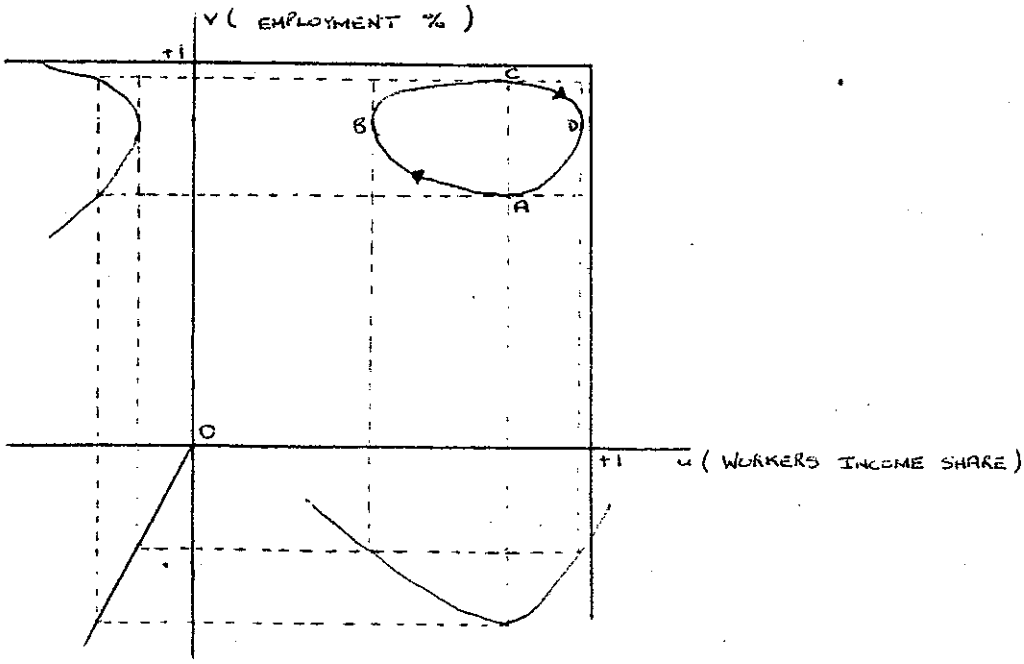
The 'natural' period will be defined by one, usually independent, variable. For example, a physical 'system' such as the weather system, is driven by an external energy source, the sun, whose effect on any one part of the globe repeats after a period of one calendar year. Thus for the weather system, which is continuous in time, the 'natural' period in which to sample the system is annually. Where the system is not affected by a periodic independent variable such as where a hydro-dynamic flow is powered by a constant heat difference, then the natural period of the system will be defined by one of the variables of the system reaching an extreme value. For example, in the hydro-dynamic flow system analysed by Lorenz (1963, p.138), the natural period was chosen as occurring when one variable (z), which represented the distortion of the temperature profile in a fluid from linearity, reached a relative maximum. There would be no reason to expect that a natural period defined in this way would consist of a constant number of units of calendar time. Thus one period, as defined by the time between relative maxima of the variable z, may last one second in calendar time and the following period might last six months.

Trajectories

The non-explosive trajectory of a set of coupled differential equations in phase space may follow several paths corresponding to different orbits of difference equations. All potential paths may be described in terms of a three dimensional phase space, with dimensions (x, y, z). If all seed values within a domain produce trajectories which converge on a single point in phase space such that in the limit the trajectory becomes a single stationary point then, at the end of each natural period, the sample point will be in the same place. A trajectory consisting of a single point (to which the trajectories from all seed values are drawn) will therefore correspond to an attractor of period one in the single dimension example given above (i.e. 0.25<A<0.75).

The second possibility is that the trajectories from all seed values will converge on a trajectory which is a closed loop in phase space. Thus, once attracted to the loop, the point which represents the 'state' of the system will continuously repeat the same path through space. Such a closed loop, which is consistent with the closed loops developed by Goodwin in two-dimensional phase space (1951, p.15 and 1965, p.57), may correspond to either a single-period or multi-period (limit cycle) attractor. In Goodwin's model (1965), the behaviour of the economy is described by a closed loop in a two dimensional phase space of employment percentage and workers' share of national income. This loop is reproduced in Figure 3.5.

<u>Figure 3.5</u>



The points A and B in Figure 3.5 identify points on the loop. If, at the end of each natural period, the point in phase space which represents the state of the economy has returned to a point such as A, then the orbit generated will be a single point, corresponding to an attractor of period one. If, however, the point alternates between points A and B from one period to the next, then the orbit generated will be an attractor, or limit cycle, of period two. A closed loop such as that in Figure 3.5 is capable, depending on the occurrence of the natural period, of generating an attractor of very high periodicy. However, as the loop is closd, it would still be periodic.

The two-dimension case, such as used by Goodwin, is restricted in that, except possibly by artificial contrivance, the point representing the 'state' of the system must eventually intersect its previous path and therefore form a closed loop.[4] In a three-dimensional system, however, it is possible for the trajectory to remain within a bounded region within which the trajectory never intersects itself. The orbit generated by such a trajectory will never repeat itself and will form an infinite number of points in space.

<u>Sensitivity Dependence on Initial Conditions</u>

An important characteristic of these non-periodic trajectories has been identifed by Lorenz (1963, p.133).

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4 The loop must be closed as each point in phase space represents a unique state of the system and thus can have only one path leading from it.

This characteristic is that, whereas a point close to a periodic trajectory is stable and will be attracted into that trajectory, points close to a non-periodic trajectory are unstable in that "two states differing by imperceptible amounts may eventually evolve into two considerably different states" (Lorenz, 1963, p.133).

This characteristic is termed sensitivity dependence on initial conditions. Systems exhibiting this behaviour are called chaotic. For examples of this behaviour, in terms of the single equation example given above, see Tables A.5 and A.6 in which initial conditions (seed values) which are close give rise, in subsequent periods, to values which differ considerably.

The characteristic of sensitivity dependence on initial conditions, as well as the fact that small variations in the shape of functions will give rise to widely varying trajectories, has led Lorenz (1963, p.141) to suggest that for systems which exhibit non-periodic behaviour, the accurate predicting of the future is impossible. This is because observations will never be precise enough to allow either the equations on which the operation of the system is based, or the 'state' of the system, to be exactly specified.

MODEL AND RESULTS

The previous three chapters have laid out the groundwork of this thesis. Chapter 1 has provided a methodological foundation for studying the economics of uncertainty in terms of the model set out by Keynes in the General Theory and without the, possibly invalidly included, alterations imposed on this model by the neo-classical revival. Chapter 2 then developed the concept of 'uncertainty' and outlined the features to be expected of a capitalist economy operating under conditions of uncertainty. One conclusion of this was that such a conceptual model would best be illustrated in terms of a non-linear difference or differential equation system. Chapter 3 then outlined the existing understanding of such models in economics and showed that the additional case of a stable and non-periodic path could exist. The characteristics of such a mathematical equation and of the path of a series of points generated by it were then set out.

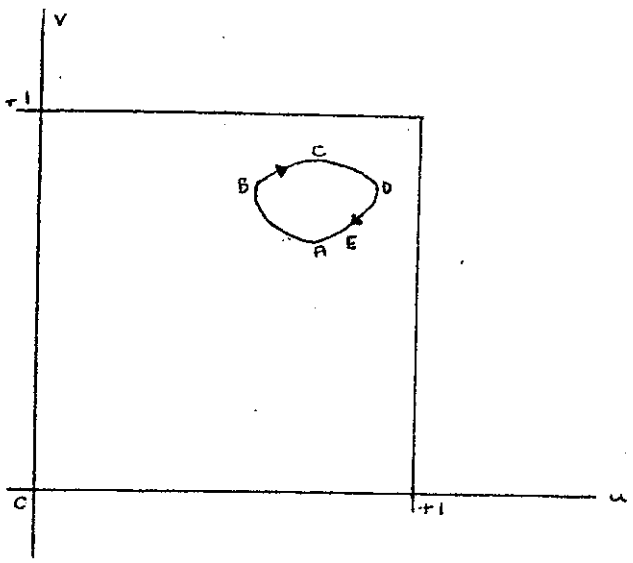
In this chapter the features outlined in Chapter 3, which are presented there in either purely mathematical form or as examples from the physical sciences, are applied to economics. This chapter then involves several sections. Firstly, the similarities between the expected features of an economy operating under conditions of uncertainty, as developed in Chapter 2, and the characteristics of mathematical systems which generate strange attractors, as developed in Chapter 3, are outlined. The conditions necessary for an economy to develop a strange attractor are then derived and the features of such an economy outlined. Finally, the implications for the role and scope of 'economics' in an economy operating under uncertainty will be considered.

In Chapter 2 it was determined that an essential feature of an economy operating under conditions of uncertainty is that it is periodic. Thus a static analysis, which necessarily excludes the influences of expectations, will be an inappropriate method of modelling such an economy. There are two types of periodic models; differential equation models utilising instantaneous time periods, and difference equation models which utilise a discrete time period. The relationship between these two types of model has been considered and it has been shown that the 'state' of the system generated by a difference equation model can be obtained from the related differential equation model by sampling the differential equation model at the end of each 'natural' period. The difference equation model, therefore, is a valid device to use in constructing a model of an economy operating under uncertainty, whether this be a specific set of mathematical equations or a more conceptual model as is being developed in this thesis.

The form of the reaction functions to expectations which will be found in an economy operating under uncertainty was discussed in Chapter 2. It was concluded there that, in a capitalist economy, the most fundamental relations would be response functions of producers (firms) in terms of output levels and the distribution of income between workers and capitalists. It was argued that Goodwin's (1965) model was conceptually identical to this expected model. The essential feature of these models is that the reaction or decision functions which determine the 'state' of the economy were folded, or non-linear, equations. The general characteristics of these functions were that they increased (decreased) smoothly to a single maximum (minimum) and then declined (increased). These characteristics are the same as for the functions analysed by Metropolis (1973) and for which Feigenbaum (1978 and 1979) subsequently established the generality of period-doubling and the decay into non-periodic orbits called strange attractors.

As is suggested above, it should be possible to analyse Goodwin's differential equation model in terms of a difference equation model. The results of Goodwin's model are summarised in the article (Goodwin, 1965, p.57) in terms of a closed loop in phase space. In this case, the phase space consists of workers' share of production in the horizontal dimension (u) and level of employment, expressed as a percentage of full employment, in the vertical dimemsion (v). This part of Goodwin's Figure 3 is reproduced here as Figure 4.1.

Figure 4.1



With this specification it would be expected that the loop would be in the upper right hand corner of the phase space. With Goodwin's differential equation system, the state of the economy will move in a clock-wise direction around the loop. Points A and C respectively represent points of minimum and maximum employment (or as a proxy to this, output), while points B and D represent minima and maxima of workers' share of output. In converting this analysis to difference equation form, it is necessary to determine a 'natural' period for the system.

Natural Period

In the article no period is defined. Many economists, and particularly econometricians, implicitly suggest that the 'natural' period to use when modelling an economy is the calendar year. This can be seen from the preponderance of the use of annual, or suitably seasonally adjusted monthly and quarterly data in the estimation of economic models. At first glance, this would appear to be a reasonable assumption, in view of the large influence that the annual seasons have on the economy. It is easily demonstrable, however, that the annual seasons cannot provide a natural period for an economy operating under conditions of uncertainty. For example, if there were no uncertainty in an economy, then the economy would always remain in a state of full employment, even though the annual driving force of the seasons was being applied. Clearly, therefore, this same driving force cannot be the basis of a 'natural' period in an economy which differs by the inclusion of an additional significant variabale. The extra significant variable which occurs in an economy operating under uncertainty, which is not present in a 'certain' economy, is the state of expectations. Thus the state of expectations, which is a deterministic and independent variable in Keynes' model, provides the basis for the definition of the natural period of the economy. In accordance with Lorenz's (1963) analysis, the 'natural' period with which to model an economy operating under conditions of uncertainty is defined by the level of expectations reaching an extreme value. Therefore, the correct period on which to base any analysis of a real economy is that period from one peak in expectations to the next or from one trough in expectations to the next.

In relating the 'expectations cycle' to the real behaviour of the economic system, it will be the case that the expectations cycle would cause, and therefore lead, the physical trade cycle. This interpretation would seem to be supported by Keynes' analysis of the trade cycle which says; "I suggest that the essential character of the trade cycle... is mainly due to the way in which (the schedule of) the marginal efficiency of capital fluctuates." (1936, p.313). The schedule of the marginal efficiency of capital depends, at least in part, on expectations as to the future yield of capital goods (1936, p.315).

In Goodwin's model, time is not introduced except in a 'logical' sense, and the functional relationships he defines are independent of the level of expectations. It would not, however, violate Goodwin's model to suggest that the 'natural' period of his analysis was one cycle of the physical trade cycle. Goodwin's and Keynes' models can therefore be shown to be essentially equivalent if the response function to employment, which Goodwin bases on actual returns, were modified to be a similarly shaped function of expected returns. This is the specification suggested by the analysis of Chapter 2 above.

Where the response functions are functions of expected returns, then the curve generated in Goodwin's analysis can be interpreted in either of two ways. Either it can remain representing the actual state of the economy in which the expectations cycle would be expected to lead the physical cycle, or Goodwin's loop could represent the point to which the economy would move if the existing state of expectations were maintained indefinitely. Both curves would, if limited to two dimensions, form a closed loop similar to that proposed by Goodwin, although the latter would be accentuated in the employment dimension relative to the points which the economy actually reaches during a trade cycle.

Expectations

If the 'state' of expectations is introduced explicitly into the model as an extra variable, and thereby as a third dimension in phase space, then, as seen in Chapter 3, the possible trajectories of the economy would be far more complicated. This is the final form of the model whose simple mathematical structure was set out in equation 2.1. That is, the 'state' of an economy operating under conditions of uncertainty can conceptually be represented by a point in phase space with the dimensions employment (%), income distribution and the 'level' of expectations.

Mathematically the form of this model is;

$$(E(t), U(t), D(t)) = f(E(t-1), U(t-1), D(t-1) \qquad (4.1)$$

It should be noted that this specification differs from Keynes' specification in that expectations are endogenised in this model, whereas Keynes identifies expectations as an independent variable. The assumption that is made here is that expectations about the future state of the economy held by each economic agent, and the degree of certainty with which those expectations are held, are based on information generated from within the economic system. Thus, the variable 'expectations' would be correctly specified as an endogenous variable. It should be noted that this specification differs from 'rational' expectations' only in that the expectations held about the future are not necessarily correct.

Goodwin's model restricts the economy to a single plane, in which expectations are held constant. Such a construction is the same as that utilised in the IS-LM analysis, and hence in the majority of Keynesian economics. The result of this is that these models necessarily produce a stable equilibrium or, in the case of Goodwin's model, an equilibrium which is stable at the end of each period. Thus, Goodwin's model would place the economy at a point such as E in Figure 4.1 at the end of each natural period where the period is defined by the relative minimum of each expectations cycle.

If the economy were allowed to be represented by a trajectory in three-dimensional space, however, as seen in Chapter 3, the possible outcomes are more complex. These outcomes can be represented in several ways. Firstly, as the trajectory in three-dimensional phase space generated by the differential analysis, secondly, as the series of points forming the orbit generated by those points of the trajectory corresponding to the end of each natural period, or thirdly, as the projection of the points forming the orbit into the employment/income distribution plane. In this third case then, Goodwin's model would be represented by a single stationary point in the employment/income distribution plane to which the economy returns at the end of each period. This would correspond to an attractor of period one. For the more complicated three-dimensional model in which the response functions are functions of expectations, the full range of attractors would potentially be generated in the employment/income distribution plane.

Trajectory of a Real Economy

The 'state' of an economy, therefore, is uniquely determined by the employment level, income distribution and the 'state' of expectations. From this analysis, the following conjecture can be made:

> The trajectory of a real economy through phase space will never intersect itself and form a closed loop. Thus the correct representation of an economy is that described by a strange attractor rather than a unique equilibrium position or a repeating cycle of equilibrium points.

This conjecture can be seen to hold by consideration of the variable 'expectations'. In the model developed above, the 'level' or 'state' of expectations has been described by a single value. This description is incomplete in that the 'state' of expectations at any one point in time encompasses the expectations of all economic agents about the state of the economy for a large (infinite) number of periods into the future. In addition, the 'state' of expectations also reflects the degree of certainty or conviction with which these expectations are held. In view of the complexity inherent in the variable 'expectations', it is extremely unlikely that the economy would ever exactly repeat itself as this would require that both the physical characteristics of the economy, in terms of employment and income distribution, and the psychological characteristics, in terms of the expected state of the economy in multiple future periods and the degree of certainty with which those expectations are held by <u>all</u> economic agents, would be <u>exactly</u> the same at two separate points in time. Such an event is so unlikely as to be, for all practical purposes, irrelevant.

Therefore, in an economy operating under conditions of uncertainty as proposed by Keynes, the only feasible path through time is one which is non-periodic and yet which is deterministic, given that expectations are endogenously determined by the flow and distribution of information generated by the economy.

<u>Comparison with Keynes' Model</u>

The above analysis sets out the workings of a capitalist economy, as described by Keynes in the General Theory, when operating under conditions of uncertainty. This model can now be compared to the summary of Keynes' model set out in Chapter 18 of the General Theory (1936, pp.245-254). In Keynes' system, the independent variables are the wage-unit, the quantity of money and the psychological factors of the propensity to consume, the schedule of the marginal efficiency of capital and the rate of interest (schedule of liquidity preference). These psychological factors are themselves dependent on expectations and the degree of uncertainty. The dependent variables are the volume of employment and the national income. The given factors or parameters consist of the quality and quantity of labour, level of technology, degree of competition, tastes, disutility of labour and the social and institutional structure.

All of these features of Keynes' model are consistent with the model developed in this thesis, with the exception of the treatment of expectations. In this model, expectations are formed endogenously on the basis of the information generated and distributed by the system's markets and institutions. It should be noted, however, that expectations remain a deterministic variable and thus this model differs fundamentally from Keynesian economics.

Necessary Conditions

In addition to the parameters noted above, it is necessary to specify two further factors as being fixed. The first of these is the impact or stance of government policy. This is necessary as a change in government policy would alter the expected profit maximising behaviour in any situation and thereby cause the underlying reaction function to alter, i.e. the effect would be to alter one of the parameters of the model.

The second additional factor which it appears necessary to specify as being fixed is related to the form of the underlying maximising function. This function, whether it be maximising profit or some other objective, is a response or reaction function to the 'state' of the economy at the end of each period. The function will therefore reflect the collective rational responses of all economic agents, in the light of the available, although incomplete, information. It is possible, of course, that for each particular 'economy', as defined by the parameters previously listed, within which economic agents find themselves, there will be a period of learning such that the particular responses of agents to any given state of the economy might differ from period to period as learning occurs. Learning, therefore, can be seen to alter the shape of the response function and therefore a requirement for a stable response function is that all learning by economic agents be completed.

Equilibrium

It should be noted that this condition is the same as that proposed by Hahn for describing an economy in equilibrium:

> "An economy is in equilibrium when it generates messages which do not cause agents to change the theories which they hold or the policies which they pursue." (1973, p.25)

What is significant is that equilibrium, as defined by Hahn, is compatable with the model developed in this thesis. In view of Hahn's analysis, therefore, it is now possible to produce a new definition of 'equilibrium' in economics:

> "An economy is in a state of equilibrium when, at

> all points in time, it lies on a trajectory in phase space which is stable in that the underlying functions are stable and which remains confined within some 'region' of that phase space."

Confinement within a region will ensure that the economy neither explodes nor leads to the extinction of the population and is therefore consistent with Keynes' observation that the economy is not violently unstable (1936, pp.249-251).

The above definition is consistent with a large range of 'states' as being positions of equilibrium. A trajectory which is a single unmoving point in phase space describes the 'stable' equilibrium positions of the Keynesian, neo-classical and General Equilibrium models. A closed loop in phase space is consistent with either a trade cycle model such as Goodwin's which returns to the same place at the end of each cycle, or one which repeats itself after a number of periods. Alternatively, a non-periodic trajectory, the projection of whose orbit into two dimensional phase space is a strange attractor, is also an 'equilibrum' state and, as argued above, it is the state most likely to be consistent with the behaviour of an economy operating under conditions of uncertainty.

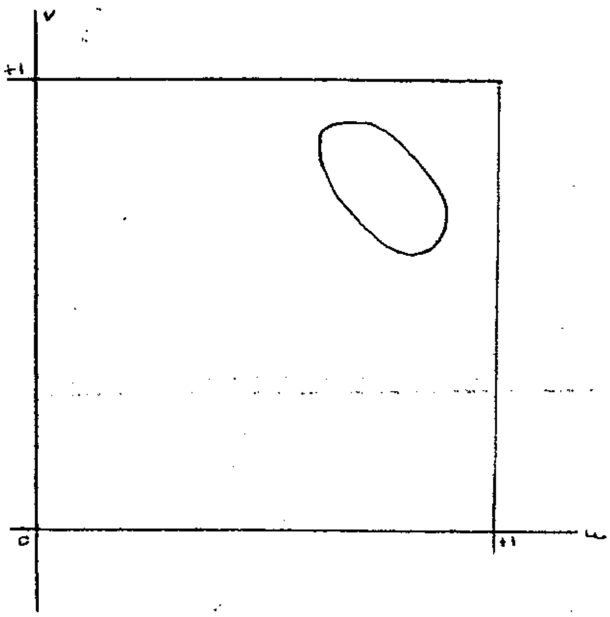
<u>Characteristics</u>

If, in fact, the behaviour of the economy is non-periodic then it will possess several characteristics which are contradictory to the usual macroeconomic models constructed by economists. The most important of these characteristics is the instability of non-periodic trajectories identified by Lorenz (1963, p.133).

Most economic theories and models assume that the economy is inherently 'stable' in that two initial conditions which are close together will remain close together and, in fact, merge into a single stable point in the long-run. If the economy is non-periodic, however, it will be unstable in that two points near each other will quickly diverge and lead to entirely different states of the economy. Thus, in order to make accurate long-term predictions as to the state of the economy, it is necessary not only to know the exact specifications of all the functions, which is impossible for all practical purposes, it is also necessary to know the exact position of the economy at some point in time. The practical impossibility of satisfying these conditions suggests that the possibility of accurate long-range forecasting of the state of the economy is non-existent (paraphrase of Lorenz, 1963,p.141). The implications of this model for the scope and role of 'economics', when considering economies operating under conditions of uncertainty, are in fact quite restricted, being limited to the very short and the very long-run behaviour of the economy. In the very short-run, it should be possible to predict the path of the economy for some time into the future with a reasonable degree of accuracy. This would be possible because the 'state' of the economy in the very near future is largely determined by the existing state of the economy and by decisions taken today on the basis of existing expectations. It would not, however, be possible to predict the state of the economy at a point of time in the future in which decisions made tomorrow or in later periods, based on information not available today, were a significant determinant of that state. Thus, while it should be possible to predict with some accuracy the immediate trajectory of the economy, it would not be possible to accurately predict that trajectory over an entire expectations cycle. For example, the economy now appears to be coming out of a recession and it should be possible to predict with some degree of accuracy the trajectory of the economy over the next few months. It would be impossible, however, to determine with any accuracy the state of the economy at the end of this next cycle, i.e. at the bottom of the next recession. Therefore, for all practical purposes, the movement of the economy from the end of one natural period to the next is indeterminant due to the impossibility of knowing the exact state of the economy at any point in time. The orbit of the economy will therefore consist of a series of points inside a region of phase space, but whose location within that region is essentially randomly generated. There is, therefore, no scope for economists attempting to predict the state of the economy in anything but the very short-run. It should be noted that the conclusion reached here is consistent with that of Blaug (1978, pp.82-83) because of practical measuring difficulties and not because the model is indeterminate.

There is scope, however, for analysing the region of phase space within which the orbit of the economy is contained. The projection of the outer edge of this region into employment/income distribution space, which would be the outer bounds of the strange attractor, would appear somewhat like the region outlined in Figure 4.2.

Figure 4.2



This region represents the outlines of the attractor generated by the natural period defined by the minimum of the expectations cycle. It is elongated in a North-West to South-East direction on the basis of the a priori expectation that a higher employment level would occur with a lower share of income going to workers. A second attractor, generated by the periods defined by the peaks in the expectations cycle, would be expected to lie generally North-East of the attractor in Figure 4.2. The attractor associated with the peak in expectations could easily overlap that of Figure 4.2, simply reflecting the fact that the relative maxima of employment in some cycles could be below the relative minima of employment in other cycles.

The location of the attractors will depend on the parameters of the system. The study of this relationship, which corresponds to the existing long-run growth theory in economics, is also a feasible role for the economist. It should be noted, however, that as changes in parameters are common in the real world, and will change the position and shape of the strange attractors, the potential for statistically examining any one attractor is also very limited.

As the location of the attractors in phase space is determined by the parameters of the system, there is no a priori reason to expect that either of the two attractors identified in Figure 4.2 will extend to include areas of phase space in which the level of unemployment falls to zero. Thus there would be no reason to expect that an economy would necessarily attain a state of full employment in the long-run, and it would almost certainly be the case that such a state, if attained, would not be maintained.

Conclusion

A major reason for the abandonment of Keynes' economics in favour of deterministic models in which the role of expectations are ignored, has been the lack, even at a conceptual level, of a method of incorporating expectations into the model in a way which retains the key features of Keynes' model and yet remains deterministic. In this thesis the characteristics of economies operating under conditions of uncertainty have been reconsidered and a model derived which, at least on a conceptual level, allows expectations to have a deterministic role while, at the same time, being consistent with Keynes' analysis. This model is based on non-linear differential and difference equations and utilises a characteristic of these equations previously unrecognised in economics.

It has been shown that because the movement of the economy through time is non-periodic, then the condition of sensitivity dependence on initial conditions will, for all practical purposes, limit the scope of macroeconomic analysis to short-run prediction and the analysis of very long-run trends. In particular, it can be seen that the predictions of macroeconomic models must diverge from reality after only a relatively short period of time due to the impossibility of accurately determining the 'state' of the world at any one moment of time. The assertion commonly made that the economy will tend, over a sufficiently long period of time, to a state of full employment equilibrium, has been shown to be incorrect in that the behaviour of an economy is almost certain to be non-periodic, in which case the economy will definitely not remain in any one state which it happens to attain and, additionally, there is no reason to believe that the state of 'full employment' will even lie within the attractors defined by the existing set of parameters.

While these results, and others suggested by the model, are both significant and of interest, the key result of this thesis has been to provide a conceptual framework which is consistent with Keynes' analysis and which incorporates expectations into economic analysis as a deterministic variable.

APPENDIX

The following tables list the first 50 results of the series generated by the iteration of the function;

$$y(t) = 4Ay(t-1)(1-y(t-1))$$

with the indicated seed values and values of the scaling variable (A).

All calculations were made on a System-80 microcomputer to 16 significant decimal places.

TABLE A.1

A:	.3	.3	.3
SEED:	.01	.999	.5
	.01188	1.1988E-03	.3
	.0140866	1.43684E-03	.252
	.0166658	1.72173E-03	.226195
	.0196657	2.06251E-03	.210037
	.0231348	2.46991E-03	.199106
	.0271195	2.95657E-03	.191355
	.0316608	3.5374E-03	.185686
	.0367901	4.22986E-03	.181448
	.0425239	5.05436E-03	.17823
	.0488587	6.03458E-03	.175757
	.0557658	7.1978E-03	.173839
	.0631872	8.57519E-03	.172343
	.0710335	.010202	.171169
	.0791853	.0121175	.170244
	.087498	.0143648	.169513
	.0958105	.0169901	.168934
	.103957	.0200417	.168475
	.11178	.0235681	.168109
	.119142	.0276152	.167818
	.125937	.0322231	.167586
	.132092	.0374217	.167401
	.137573	.0432256	.167254
	.142376	.0496286	.167136
	.146526	.0565987	.167042
	.150067	.0640743	.166967
	.153056	.0719626	.166907
	.155556	.0801407	.166859
	.15763	.0884618	.16682
	.159339	.0967636	.166789
	.16074	.104881	.166765
	.161884	.112657	.166745
	.162813	.119958	.166729
	.163566	.126682	.166717
	.164174	.13276	.166707
	.164665	.138162	.166699
	.165061	.142888	.166692
	.165379	.146965	.166687
	.165634	.15044	.166683
	.16584	.153369	.16668
	.166004	.155816	.166677
	.166136	.157845	.166675
	.166242	.159516	.166673
	.166327	.160885	.166672
	.166395	.162001	.166671
	.166449	.162908	.16667
	.166492	.163643	.166669
	.166527	.164237	.166669
	.166555	.164716	.166668
	.166577	.165101	.166668
	.166595	.165411	.166668
		.165661	

A :	.7	.7	.7
SEED:	.01	.999	.5
	.02772	2.7972E-03	.7
	.0754645	7.81025E-03	.588
	.195355	.0216979	.678317
	.440136	.0594359	.610969
	.689966	.156529	.665521
	.598957	.369678	.623288
	.672581	.652445	.65744
	.616604	.634929	.630595
	.66193	.649023	.652246
	.626581	.637818	.6351
	.655137	.646818	.648895
	.632611	.639645	.637925
	.65076	.645398	.646735
	.63636	.640806	.639713
	.647937	.644486	.645345
	.638721	.641547	.64085
	.646118	.643901	.644452
	.640219	.642019	.641574
	.644948	.643526	.643879
	.641172	.642321	.642037
	.644197	.643285	.643512
	.64178	.642514	.642333
	.643716	.643131	.643276
	.642168	.642638	.642522
	.643407	.643033	.643125
	.642416	.642717	.642643
	.643209	.642969	.643029
	.642575	.642767	.64272
	.643082	.642929	.642967
	.642677	.6428	.642769
	.643001	.642903	.642928
	.642742	.64282	.642801
	.64295	.642887	.642902
	.642783	.642834	.642821
	.642916	.642876	.642886
	.64281	.642842	.642834
	.642895	.642869	.642876
	.642827	.642848	.642842
	.642881	.642865	.642869
	.642838	.642851	.642848
	.642873	.642862	.642865
	.642845	.642853	.642851
	.642867	.64286	.642862
	.642849	.642855	.642853
	.642864	.642859	.64286
	.642852	.642856	.642855
	.642861	.642858	.642859
	.642854	.642856	.642856
	.64286	.642858	.642858
	.642855	.642857	.642856

TABLE A.3

A:	.785	.785	.785
SEED:	.01	.999	.5
	.031086	3.13686E-03	.785
	.0945757	9.81884E-03	.529954
	.268882	.0305284	.782183
	.617275	.0929329	.534971
	.741814	.264691	.78116
	.601391	.611137	.53678
	.75272	.746217	.780752
	.584456	.594645	.537499
	.762603	.756873	.780585
	.568465	.577811	.537795
	.770282	.765989	.780515
	.555616	.562845	.537918
	.775287	.772599	.780485
	.547041	.551667	.53797
	.778052	.776618	.780473
	.542238	.544735	.537992
	.779398	.778716	.780468
	.539881	.541076	.538001
	.780006	.779702	.780466
	.538814	.539348	.538004
	.78027	.780139	.780465
	.53835	.53858	.538006
	.780382	.780326	.780464
	.538152	.53825	.538007
	.78043	.780406	.780464
	.538068	.53811	.538007
	.78045	.78044	.780464
	.538033	.53805	.538007
	.780458	.780454	.780464
	.538018	.538025	.538007
	.780462	.78046	.780464
	.538012	.538015	.538007
	.780463	.780462	.780464
	.538009	.538011	.538007
	.780464	.780463	.780464
	.538008	.538009	.538007
	.780464	.780464	.780464
	.538008	.538008	.538007
	.780464	.780464	.780464
	.538007	.538007	.538007
	.780464	.780464	.780464
	.538007	.538007	.538007
	.780464	.780464	.780464
	.538007	.538007	.538007
	.780464	.780464	.780464
	.538007	.538007	.538007
	.780464	.780464	.780464
	.538007	.538007	.538007
	.780464	.780464	.780464
	.538007	.538007	.538007

A: .87	.87	.87
SEED: .01	.999	.5
.034452	3.47652E-03	.87
.115762	.0120562	.393588
.356218	.0414499	.830594
.798057	.138267	.489662
.560844	.414638	.869628
.857117	.844643	.394545
.426187	.456651	.8313
.851039	.863461	.488036
.441164	.410279	.869502
.857954	.841987	.39487
.424105	.462997	.831538
.849955	.865235	.487488
.443809	.40578	.869455
.859012	.839106	.39499
.421464	.469824	.831626
.848536	.866831	.487285
.44726	.401714	.869437
.86032	.836383	.395036
.418189	.476227	.831659
.846708	.868033	.487208
.451681	.398639	.869431
.861875	.834247	.395053
.414281	.481212	.831672
.84443	.868772	.487178
.457161	.396746	.869428
.863614	.832899	.39506
.409893	.484342	.831677
.841745	.869147	.487167
.463573	.395783	.869427
.865382	.832203	.395063
.405405	.485952	.831679
.838861	.869313	.487162
.470404	.395355	.869427
.866952	.831892	.395064
.401405	.48667	.83168
.836171	.869382	.48716
.476721	.395179	.869426
.868114	.831764	.395064
.398432	.486966	.83168
.8341	.869409	.48716
.481552	.395109	.869426
.868816	.831713	.395064
.396633	.487084	.83168
.832817	.86942	.48716
.48453	.395082	.869426
.869167	.831693	.395065
.39573	.48713	.83168
.832165	.869424	.487159
.48604	.395071	.869426
.869322	.831685	.395065

TABLE A.5

A : .9	.9	.9
SEED: .29	.3	.31
.74124	.756	.77004
.690492	.664071	.637482
.769367	.803091	.831955
.63879	.569289	.503301
.830654	.882717	.899961
.506404	.3727	.324113
.899852	.841661	.78863
.324425	.479763	.600095
.789025	.898526	.863932
.599273	.328238	.423193
.864521	.793792	.878763
.421647	.58927	.38354
.877899	.871311	.851173
.385892	.403661	.456038
.853126	.866588	.893043
.451088	.416209	.343863
.891387	.874725	.812237
.348537	.394494	.54903
.817413	.859926	.891346
.537297	.433631	.348654
.894992	.884143	.81754
.338332	.368764	.537006
.805909	.837998	.89507
.563111	.488727	.338111
.885661	.899543	.805651
.364556	.325317	.56368
.833957	.790149	.885402
.498501	.596929	.365276
.899992	.866177	.834658
.324023	.417292	.496814
.788516	.875374	.899964
.600331	.392741	.324105
.863761	.858584	.78862
.42364	.437104	.600115
.879009	.885759	.863917
.382869	.364284	.423232
.850609	.833693	.878784
.457464	.499137	.383482
.893487	.899997	.851125
.342606	.324008	.456161
.810818	.788496	.893081
.552212	.600372	.343754
.890186	.863732	.812113
.351918	.423717	.549307
.821058	.879051	.891248
.528918	.382752	.348931
.896989	.850511	.817841
.332638	.457712	.536318
.799164	.893562	.895252
.577804	.342392	.337594

TABLE A.6

A:	.95	.95	.95
SEED:	.69	.7	.71
	.81282	.798	.78242
	.578146	.612545	.646908
	.926794	.901868	.867989
	.257817	.336308	.435421
	.727121	.848179	.934152
	.753982	.489331	.233745
	.704875	.949568	.680612
	.7905	.181979	.826042
	.629317	.565677	.546048
	.886453	.933609	.941943
	.382487	.235537	.20781
	.897524	.684225	.625575
	.349503	.821033	.890078
	.863932	.558364	.37179
	.446702	.937056	.887536
	.939206	.224133	.379301
	.216974	.66081	.89464
	.645606	.851733	.358185
	.869436	.47988	.873576
	.431366	.948462	.419677
	.9321	.185752	.925483
	.240501	.574743	.262064
	.694109	.928771	.734869
	.806822	.251391	.740378
	.592269	.715135	.730429
	.917649	.774125	.748229
	.287164	.664452	.715853
	.777863	.847232	.772949
	.65661	.491835	.666896
	.856799	.949747	.844153
	.46624	.181366	.499922
	.945669	.564196	.95
	.195241	.93434	.1805
	.597062	.233125	.562095
	.9142	.679356	.935348
	.298066	.827759	.229794
	.795047	.54178	.672558
	.6192	.943367	.83685
	.896007	.203018	.518821
	.354078	.614847	.948654
	.869086	.899878	.185097
	.432347	.34237	.573177
	.932608	.855581	.929652
	.238833	.469537	.248518
	.690808	.946474	.709676
	.81165	.192513	.782936
	.580922	.590718	.645799
	.925117	.918727	.869222
	.263249	.283736	.431965
	.737006	.772274	.932411

BIBLIOGRAPHY

Argy, V. (1981), The Postwar International Crisis: An Analysis, George Allen & Unwin, London.

Baumol, W.J. (1970), Economic Dynamics an Introduction, Macmillan, U.S.A.

Benassay, J.P. (1975), 'Neo-Keynesian Disequilbrium Theory in a Monetary Economy', Review of Economic Studies, Vol.XLII(4), No.32, pp.503-523.

Blaug, M. (1978), Economic Theory in Retrospect, Cambridge University Press, Cambridge.

Boland, L.A. (1979), 'A Critique of Friedman's Critics', Journal of Economic Literature, XVII, pp.503-522.

Chick, V. (1978), 'The Nature of the Keynesian Revolution: A Reassessment', Australian Economic Papers, Vol.17, No.30, pp.1-20.

Colander, D.C. and Guthrie, R.S. (1981), 'Great Expectations: What the Dickens do "Rational Expectations" mean?', Journal of Post Keynesian Economics, Vol.III, No.2, pp.219-234.

Davidson, P. (1972), 'Money and the Real World', Macmillan, U.S.A.

Davidson, P. & Kregel, J.A., 'Keynes' Paradigm: A Theoretical Framework for Monetary Analysis', Publication unknown, Reserve Bank of Australia Research Library, p.330.1.

Dean, E. (1965), The Controversy Over the Quantity Theory of Money, D.C. Heath & Company, Boston.

Dillard, D. (1958), The Economics of J.M. Keynes, Crosby Lockwood & Son Ltd, London.

Dornbush, R. and Fisher, S. (1978), Macroeconomics, International Student Edition, McGraw-Hill International Book Company, Japan.

Eisner, R. (1975), 'The Keynesian Revolution Reconsidered', American Economic Review, Vol.5, pp.189-194.

Feigenbaum, M.J. (1978) 'Quantitative Universality for a Class of Non-linear Transformations', Journal of Statistical Physics, 19, No.1, pp.25-52.

________ (1979), 'The Universal Metric Properties of Non-linear Transformations', Journal of Statistical Physics, 21, No.6, pp.669-706.

Franceschini, V. & Tebalbi, C. (1979), 'Sequences of Infinite Bifurcations and Turbulence in a Five-Mode Truncation of the Navier-Stokes Equations', Journal of Statistical Physics, Vol.21, No.6, pp.707-726.

Friedman, M. (1953), 'The Methodology of Positive Economics', in Essays in Positive Economics, University of Chicago Press, Chicago, pp.3-43.

________ (1956), 'The Quantity Theory of Money - A Restatement', reprinted in Studies in the Quantity Theory of Money, M. Friedman (ed), University of Chicago Press, pp.3-21.

________ (1958), 'The Supply of Money and Changes in Prices and Output', U.S. Congress Joint Economic Committee, The Relationship of Prices to Economic

Stability and Growth: Compendium of Papers Submitted by Panelists, Washington, pp.241-256. Reprinted in Dean (1965), pp.87-107.

________ (1968), 'Money: Quantity Theory', _International Encyclopedia of the Social Sciences_, Free Press, pp.432-447.

Goodhart, C.A.E. (1975), _Money Information and Uncertainty_, Macmillan Press Ltd.

Goodwin, R.M. (1951), 'The Non-linear Accelerator and the Persistence of Business Cycles', _Econometrica_, 19, No.1, pp.1-17.

________ (1965), 'A Growth Cycle', Presented as a paper at the First World Congress, Rome, 1965. Reprinted in _Socialism, Capitalism and Economics Growth_, C.H. Feinstein (ed.), Cambridge University Press, Cambridge, 1967. All references are to the reprint.

Hahn, F.H. (1973), _On The Notion of Equilibrium in Economics_, Inaugural Lecture, Cambridge University Press.

________ (1982), _Money and Inflation_, Basil Blackwell, Oxford.

Hansen, A. (1953), _A Guide to Keynes_, McGraw-Hill Book Company Inc., U.S.A.

Henon, M. (1976), 'A Two-dimensional Mapping with a Strange Attractor', _Communications in Mathematical Physics_, Vol.50, pp.69-77.

Hicks, J.R. (1937), 'Mr Keynes and the 'Classics': A Suggested Interpretation', Econometrica, Vol.V, pp.147-159.

Hofstadter, D.R. (1981), 'Metamagical Themes; Strange Attractors: Mathematical Patterns Delicately Poised Between Order and Chaos', Scientific America, 245, No.5, pp.16-29.

Johnson, H.G. (1961), 'The General Theory after Twenty Five Years', American Economic Review, Vol.51, pp.1-17.

Katona, G. (1980), 'Rational Expectations? How Expectations are Really Formed', Challenge, November-December, Vol.23, No.5, pp.32-35.

Keynes, J.M. (1936), The General Theory of Employment, Interest and Money, The Collected Writings of John Maynard Keynes, D. Moggridge (ed), Vol.VII, Macmillan, 1978, Cambridge.

———— (1937), 'The General Theory of Employment', Quarterly Journal of Economics, February; Reprinted in Keynes, 1973b, pp.109-123. All references are to the reprint.

———— (1973a), The General Theory and After: Part I Preparation, The Collected Writings of John Maynard Keynes, D. Moggridge (ed), Vol.XIII, Macmillan, Cambridge.

———— (1973b), The General Theory and After: Part II Defence and Development, The Collected Writings of John Maynard Keynes, D. Moggridge (ed), Vol.XIV, Macmillan, Cambridge.

________ (1979), The General Theory and After: A Supplement, The Collected Writings of John Maynard Keynes, D. Moggridge (ed), Vol.XXIX, Macmillan, Cambridge.

Knight, F.H. (1935), Risk, Uncertainty and Profit, London School of Economic and Political Science, No.16, Houghton Mifflin Company, Great Britain.

Laidler, D. (1980), 'Monetarism: An Interpretation and Assessment', Reserve Bank of Australia Research Discussion Paper, No.8009.

Leijonhufvud, A. (1968), On Keynesian Economics and the Economics of Keynes: A Study in Monetary Theory, Oxford University Press.

Lekachman, R. (ed.) (1964), Keynes' General Theory: Reports of Three Decades, St. Martins Press, New York.

Lorenz, E.N. (1963), 'Deterministic Non-periodic Flow', Journal of Atmospheric Sciences, 20, pp.130-141.

Mason, W.E. (1981, 'Some Negative Thoughts on Friedman's Positive Economics', Journal of Post Keynesian Economics, Vol.III, No.2, pp.235-255.

May, R.M. (1976), 'Simple Mathematic Models with Very Complicated Dynamics', Nature, 261, pp.459-467.

Mayer, T. (1978), The Structue of Monetarism, W.W. Norton & Co., U.S.A.

Metropolis, N., Stein, M.L. and Stein, P.R. (1973), 'On Finite Limit Sets for Transformations on the Unit Interval', Journal of Combinatorial Theory, (A)15, pp.25-44.

Milgate, M. (1977), 'Keynes on the 'Classical' Theory of Interest', Cambridge Journal of Economics, Vol.1, No.3, pp.307-315.

Minsky, H.P. (1975), John Maynard Keynes, Columbia University Press, U.S.A.

Muth, J. (1961), 'Rational Expectations and the Theory of Price Movements', Econometrica, Vol.29, pp.315-335.

Rothschild, M. & Stiglitz, J.E. (1970), 'Increasing Risk I: A Definition', Journal of Economic Theory, No.2, pp.225-243.

________ (1971), 'Increasing Risk II: Its Economic Consequences', Journal of Economic Theory, No.3, pp.66-84.

Samuelson, P.A. (1963), Foundations of Economic Analysis, Harvard University Press, Cambridge, U.S.A.

Sargent, T. and Wallace, N. (1975), 'Rational Expectations, the Optimal Monetary Instrument and the Optimal Money Supply Rule', Journal of Political Economy, Vol.83, pp.241-254.

Shackle, G.L.S. (1949), Expectations in Economics, Cambridge University Press, Great Britain.

________ (1955), Uncertainty in Economics and Other Reflections, Cambridge University Press.

Simmel, G. (1907), _Philosophie des Geldes_, translation by T. Bottomore and D. Frisby published as _The Philosophy of Money_, Routledge & Kegan Paul, Great Britain, 1978.

Smith, P.R. (1980), 'Liquidity Preference versus Loanable Funds: A Brief Revival', _Australian Economic Papers_, Vol.19, No.34, pp.215-218.

Tobin, J. (1961), 'Money, Capital and Other Stores of Value', _American Economic Review_, Vol.51, pp.26-37. Reprinted in Dean (1965), pp.107-120.